Of Art & Math:
Introducing Ambigrams

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Mathematicians love puzzles—they love to play with numbers and shapes but often their love can turn to words and other areas that, at least on the surface, have little to do with mathematics. In this article we are going to focus on a very specific kind of artistic wordplay (and its relationship to mathematics) called *ambigrams*. The word ambigram was coined by cognitive scientist Douglas Hofstadter from 'ambi' which suggests *ambiguous* and 'gram' for *letter*. Ambigrams exploit *how* words are written and bring together the mathematics of symmetry, the elegance of typography *and* the psychology of visual perception to create surprising, artistic designs. Most of all, they are great fun!

All right, let's start with the example in Figure 1. Can you read it?

Figure 1. A 180-degree rotation ambigram for the word "Wordplay"

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Rotating the page you are holding will reveal something interesting. The word stays the same! In other words, it has rotational symmetry.

Thus ambigrams are a way of writing words such that they can be read or interpreted in more than one way. Figure 2 is another one, an ambigram for the word "ambigram."

Figure 2. A 180-degree rotation ambigram for "ambigram"

Incidentally, you may have noticed something interesting in these two examples. In the "wordplay" design each letter of the first half of the word maps onto *one letter* (w to y, o to a, and so on). Some transformations are straightforward (as in the "d" becoming a "p") while others need some level of distortion to work visually (the w-y being the most obvious example). This distortion of course is constrained since whatever shape you come up with has to be readable as specific letters in two different orientations.

Now consider Figure 2, the design for the word "ambigram." There is a lot more distortion going on here. The "stroke" that emerges from the "a" becomes the third leg of the "m." More interesting is how the "m" after the "a" actually maps onto two letters ("r" and "a") when rotated. Isn't it interesting to see that what looks like *one* letter becomes *two* when rotated? On a different note, the g-b transformation is of particular interest to the authors! Can you guess why?

Given that ambigrams work because of the specific mappings of letters (either individually or in groups) to each other implies, that even one change in the letters of the word can lead to a very different design. Thus the solution for the word "ambigrams" (plural) is quite different from the solution for "ambigram" (singular). Note how in Figure 3, many of the mappings have shifted, and the natural "g-b" transformation that made so much sense in the design for "ambigram" has now shifted to a "b-a" transformation while "g" now maps onto itself.

Figure 3.The first of two ambigrams, for "ambigrams." This design reads the same when rotated 180-degrees.

Another important aspect of *why* ambigrams work can be seen in Figure 3. Notice the initial "A" and the final "S." In the case of the "A" the gap at the bottom looks exactly like what it is, a gap. On the other hand, when rotated 180 degrees, our mind imagines a connection across this gap – to make the topmost stroke of the "S." How cool is that!

Rotation is not the only way one can create ambigrams. Figure 4 is another design for the word "ambigrams" –this time as a reflection. This design has bi-lateral symmetry (a symmetry most often found in living things – such as faces, leaves and butterflies). If you place a mirror—perpendicular to the page—in the middle of the 'g', the right half of the design will reflect to become the left part of the word.

Figure 4. Another ambigram for "ambigrams" this time with bilateral symmetry

Designs such as these read the same from right to left. This is a feature of a Palindrome. A palindrome is a word or a sentence that reads the same forwards and backwards. For example, some believe that the first sentence ever spoken was:

Madam, I'm Adam

Notably the response to this palindrome was also a single word palindrome:

Eve

Even longer examples of palindromes can be created. Here is, for instance, a palindromic poem.

I think
a lot
about
very little
and care
very little
about
a lot
I think

Reverse the sequence of lines (from bottom to top) and you will have the same poem! Here the palindrome is at the level of a line of the poem. The first line is "I think" and so is the last line. Similarly, the second line from both top and bottom is "a lot". The poem is symmetric about the phrase "and care" which comes in the middle of the poem. The symmetry is very similar to the mirror symmetry mentioned earlier, though not quite the same.

Limiting ourselves to just mirror symmetry, we can find many examples of its relevance to mathematics. For example, consider an isosceles triangle, a triangle with two sides equal. It has the same symmetry as the design above.

It is possible to prove that the base angles of an isosceles triangle are equal, just by exploiting this mirror symmetry? Here is a hint:

An Isosceles Triangle
Looks in the mirror
And finds itself un-reversed.
"My base angles are equal"
it says.

Visually this can be represented as a triangle-ambigram for the word "isosceles", see Figure 5.

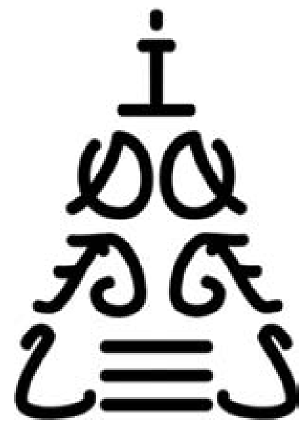

Figure 5. An isosceles triangle that reads "isosceles" when reflected in a mirror

Different types of ambigrams

Every ambigram design need not read the *same* word when rotated and/or reflected. Figure 6 is a design that reads "darpan" (the Hindi word for mirror), and "mirror" (the English word for darpan) when rotated 180 degrees.

Figure 6.The word "darpan" (hindi for mirror) becomes "mirror" on rotation by 180-degrees

So far we have seen ambigrams with a vertical line of symmetry like the designs for "ambigrams" or "isosceles" having a vertical line of symmetry. Hofstadter has called this a "wall reflection." The other is a "lake reflection" such as the example in Figure 7 – where the word "abhikalpa" (the Sanskrit word for architect) which has a horizontal line of symmetry. Mathematically speaking, a wall-reflection is a reflection across the "y-axis" while a "lake-reflection" is a reflection across the "x-axis."

Figure 7. Ambigram for "abhikalpa", an example of a lake reflection

Incidentally, the use of Hindi words in the above two designs brings up an interesting challenge. Is it possible to create an ambigram that can be read in two different languages? Here is the Sanskrit sound "Om" as traditionally written in Devanagri script. This design if rotated 90-degrees magically transforms into the letters "Om" in English!

Figure 8. The Sanskrit word "om"

Figure 9. The English "om" formed by rotating the Sanskrit "om" by 90-degrees.

Not all reflection ambigrams have to be reflected across the x- or y-axes. Consider this design (Figure 10), where the word "right" when reflected across the 45-degree axis reads "angle." (This design was inspired by a solution first put forth by Bryce Herdt).

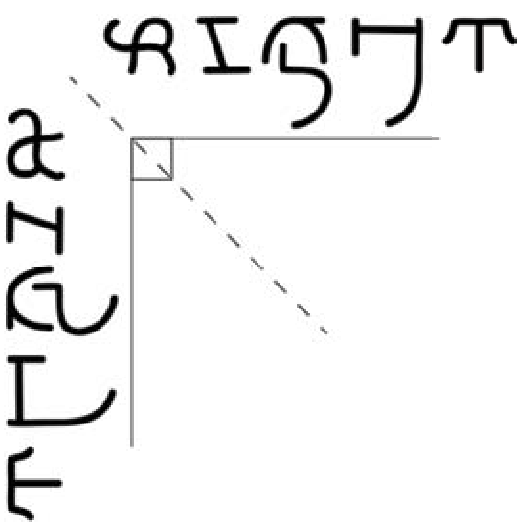

Figure 10. A special "Right angle" made specially for this special magazine

Those who are familiar with tessellations will like the next kind of designs—space-filling ambigrams. See for instance Figure 11, this design for the word "space" – where replications of the word form a network that cover a surface – in this case the surface of a sphere.

Figure 11. A space-filling ambigram for "space"

Here is an example of a rotational chain ambigram for the word "mathematics." In chain-ambigrams a word is broken into two parts – each of which maps to itself. In Figure 12 "math" maps onto itself and the rest of the word "ematics" maps onto itself.

Figure 12. An ambigram for "mathematics"

Effective chain-ambigrams can be quite rich in meaning. Consider Figure 13. This example of a chain ambigram for "action-re-action" where the letters "-re-" switch loyalty depending on whether you are reading the top part of the circle or the bottom.

Figure 13. Ambigram for "Action-re-action"

Given this idea of breaking words into shorter ambigramable pieces, it is easy to create such chain-reflection ambigrams as well—such as Figure 14 for the word "reflect." This design will read the same when you hold it up against a mirror (or peer at it from the other side of the page holding it up to a light).

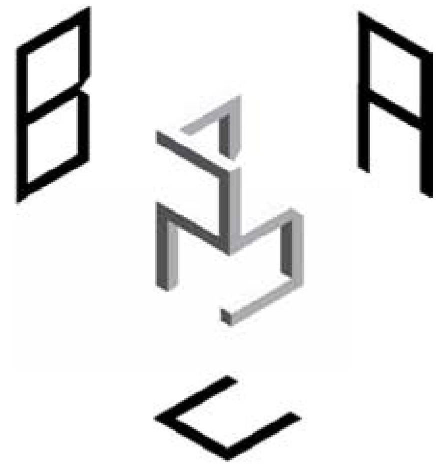

Figure 16. A triplet ambigram for "A","B" and"C"

Even seeing patterns in parts of a word can lead to interesting designs, such as the star-shaped design in Figure 17 for the word Astronomy. In designs like this one takes advantage of specific letters to

Figure 14. A chain-reflection ambigram for "reflect"

A couple of other types of ambigrams are called "figure-ground" ambigrams and "triplets." A figure-ground ambigram is akin to a tessellation – where the space between the letters of a word can be read as another word altogether. What do you see in Figure 15? Good? Evil? Can you see both? Can you see both at the same time? A good pun-ya?

Figure 15. A Figure-Ground ambigram for "Good" and "Evil"

Mathematicians who love solid geometry will love triplets! A triplet is 3-dimensional shape designed in such a way that it casts different shadows depending on where you shine light on it. For instance the design below (Figure 16) is a shape that allows you to see the letters "A," "B" and "C" depending on where you shine light on it.

create visually attractive designs. The designer in this case noted that the letter "R" could be rotated 60-degrees to make the letter "N."

Figure 17. A star shaped ambigram for "astronomy"

Aesthetics, ambigrams & mathematics

Some mathematicians speak of what they do in aesthetic terms. The famous mathematician George Polya remarked: "Beauty in mathematics is seeing the truth without effort." This mirrors

Keat's famous line "Beauty is truth, truth beauty." As Bertrand Russell said, "Mathematics, rightly viewed, possesses not only truth, but supreme beauty." Figure 18 attempts to capture this idea.

Figure 18. A design for "truth & beauty", where Beauty becomes Truth & Truth becomes Beauty.

When mathematicians speak of beauty they usually talk of theorems or proofs that are elegant, surprising, or parsimonious. They speak of "deep" theorems. Mathematical insights that are not obvious, but explained properly seem inevitable. Finally mathematicians delight in doing mathematics, which often means solving problems set by themselves or by other mathematicians.

Effective ambigram designers, in small ways, see the creating of ambigrams as sharing many of these characteristics that mathematicians speak of. The creation of ambigrams can be a highly engaging activity that can lead to seemingly inevitable and yet surprising and elegant solutions. In that sense, both mathematicans and ambigram-artists engage in what we have called "Deep Play" (DP) – a creative, open-ended engagement with ideas through manipulating abstract symbols. We must admit, however, that our teachers have often considered what we do as being more TP (Time Pass) than DP (Deep Play!). We hope we have been able to give you some of the flavor of the art and mathematics of ambigrams. In subsequent articles we will delve deeper into the mathematical aspects of these typographical designs, and use them to communicate mathematical ideas such as symmetry, paradoxes, limits, infinities and much more.

About the authors

PUNYA MISHRA, when not creating ambigrams, is professor of educational technology at Michigan State University. GAURAV BHATNAGAR, when not teaching or doing mathematics, is Senior Vice-President at Educomp Solutions Ltd.

Loving both math and art, Punya's and Gaurav's collaboration began over 30 years ago when they were students in high-school. Since then, they have individually or collectively, subjected their friends, family, classmates, and students to a never ending stream of bad jokes, puns, nonsense verse and other forms of deep play. To their eternal puzzlement, their talents have not always been appreciated by their teachers (or other authority figures). Punya's email address is punya@msu.edu and his website is at http://punyamishra.com. Gaurav's email address is bhatnagarg@gmail.com and his website is at http://gbhatnagar.com

All the ambigrams presented in this article are original designs created by Punya Mishra (unless otherwise specified). Please contact him if you need to use them in your own work.

You should note, dear reader, that Punya and Gaurav have hidden a secret message in this article. If you can work out what it is, or if you have any input, thoughts, comments, original ambigram designs to share, please drop them a note at their e-mail IDs.